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LETTER TO THE EDITOR

Comments on "Adiabatic steam–water annular flow in an annular geometry" by P. S. Andersen and J. Würtz

Andersen & Würtz (1980) have concluded that the entrainment rates per unit area from the two surfaces of an annulus are equal. This seemed very surprising and so the assumptions behind this paper have been examined in detail.

The analysis below starts by examining when the conclusion that $E_1 = E_2$ is likely to be valid. Define E_1 = total entrainment rate from surface 1 per unit area, E_2 = total entrainment rate from surface 2 per unit area (figure 1). Then net loss from surface 1 per unit length = A_1E_1

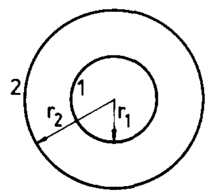

Figure 1.

$F_{12} - A_2E_2F_{21}$ where F_{ij} = fraction of particles leaving surface i which hit surface j and A_i = surface area of surface i per unit length. At hydrodynamic equilibrium this net loss is zero, and so

$$A_1\, E_1\, F_{12} - A_2\, E_2\, F_{21} = 0\,. \qquad [1]$$

From Hottel & Sarofim (1967)

$$A_1\, F_{12} = A_2\, F_{21} \qquad [2]$$

hence

$$E_1 = E_2\,. \qquad [3]$$

Actual values of the view factors F_{ij} can be calculated. By geometry $F_{12} = 1$ (all particles leaving surface 1 and travelling in a straight line must hit surface 2). Therefore

$$F_{21} = \frac{A_1F_{12}}{A_2} = \frac{r_1}{r_2}\,. \qquad [4]$$

Since

$$\sum_{\text{all } i} F_{ij} = 1 \qquad [5]$$

then

$$F_{11} + F_{12} = 1 \qquad [6]$$

and

$$F_{21} + F_{22} = 1 \tag{7}$$

and therefore

$$F_{11} = 0 \tag{8}$$

and

$$F_{12} = 1 - \frac{r_1}{r_2}. \tag{9}$$

Looking in more detail at these view factors and the assumptions underlying them, consider F_{21} in detail (figure 2).

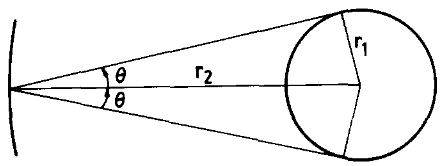

Figure 2.

$$F_{21} = \frac{\int_{-\theta}^{+\theta} \cos\theta \, d\theta}{\int_{-\pi/2}^{+\pi/2} \cos\theta \, d\theta}, \theta = \sin^{-1}\frac{r_1}{r_2} \tag{10}$$

$$F_{21} = \frac{\left[\sin\theta\right]_{-\sin^{-1}(r_1/r)}^{+\sin^{-1}(r_1/r_2)}}{\left[\sin\theta\right]_{-\pi/2}^{+\pi/2}} = \frac{r_1}{r_2} \text{ as before.} \tag{11}$$

The cos θ term arises because of Lamberts law for radiating surfaces. Thus it can be seen that thermal radiation theory leads inevitably to the conclusion that for fully developed flow E_1 and E_2 are equal. When the process of entrainment is considered in more detail doubt can be cast upon this strict application of radiation techniques.

The entrainment takes place at or near the tips of the waves on the liquid film in annular flow, see for example Whalley *et al.* (1979). The entrainment process is explosive and appears to project droplets in all directions—a few are even projected *back* towards the wall. If it is assumed that drops are scattered uniformly in all forward directions, then a modified view factor F'_{21} can be defined as

$$F'_{21} = \frac{\int_{-\theta}^{+\theta} d\theta}{\int_{-\pi/2}^{+\pi/2} d\theta} = \frac{2}{\pi}\sin^{-1}\frac{r_1}{r_2}. \tag{12}$$

From [1], but now using the modified view factors

$$\frac{E_1}{E_2} = \frac{F'_{21}A_2}{F'_{12}A_1}. \tag{13}$$

As above $F'_{12} = 1$ because all drops leaving the vicinity of surface 1 must hit the surface 2 and so

$$\frac{E_1}{E_2} = \left[\frac{2}{\pi}\sin^{-1}\frac{r_1}{r_2}\right]\frac{A_2}{A_1}. \qquad [14]$$

In this case $r_1 = 17/2$ mm and $r_2 = 26/2$ mm and so

$$\frac{E_1}{E_2} = \left[\frac{2}{\pi}\sin^{-1}\frac{17}{26}\right]\cdot\frac{26}{17} = 0.694\,. \qquad [15]$$

Thus the entrainment rate per unit area from the rod should be less than that from the outer tube. If the entrainment rate is characterised by the group $\tau_i m/\sigma$, see Hutchinson and Whalley (1973), where τ_i = interfacial shear stress, m = average film thickness and σ = surface tension. τ_i will depend on the gas + entrained liquid flow-rate in the gas core and also on the liquid film thickness. Hence if the entrainment rate is greater for the outer tube, then the liquid film thickness and therefore the liquid film flow-rate per unit periphery ($W_F/2\pi r$) must also be greater. The results in table 1 of Andersen & Würtz (1980) substantiate this conclusion.

Thus it can be demonstrated that the data presented support an explosive mechanism for droplet entrainment which shoots out droplets equally in all directions.

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